# EFFICIENT ADMITTANCE MATRIX REPRESENTATION OF A CUBIC JUNCTION OF RECTANGULAR WAVEGUIDES

**V.E. Boria, M. Guglielmi***
**Departamento de Comunicaciones, Universidad Politécnica de Valencia**
**Camino de Vera s/n, 46071 Valencia, Spain**
***European Space Research and Technology Centre**
**PO Box 299, Noordwijk, The Netherlands**

## ABSTRACT

In this paper we describe a very efficient multimode admittance matrix representation of a six-port *Cubic* junction composed of the orthogonal intersection of three rectangular waveguides. The formulation is based on the theory of cavities and yelds very simple closed-form analytical expressions for all matrix elements. In addition to theory, the application of the results obtained to the special cases of E- and H-plane T-junctions and to the Magic-T junction are also discussed indicating how the representations obtained are indeed very accurate and numerically efficient.

## 1. INTRODUCTION

Rectangular waveguide junctions are often used in several passive components like, for instance, orhtomode transducers, multiplexers, and couplers. The development of fast and accurate algorithms for their analysis has therefore received considerable attention in the literature [1]-[3].

In this paper we present an efficient multimode admittance matrix representation for the six-port device in Fig. 1, called the *Cubic* or C-junction. The analysis is based on the theory of cavities [4], and yelds closed-form analytical expressions for the admittance matrix elements which are practically independent from frequency.

In addition, we also discuss how the general matrix derived can be used to study common junctions like E- and H-plane T-junctions, and Magic-T junctions. Comparisons with available data are shown indicating how the representations derived are indeed very accurate and efficient.

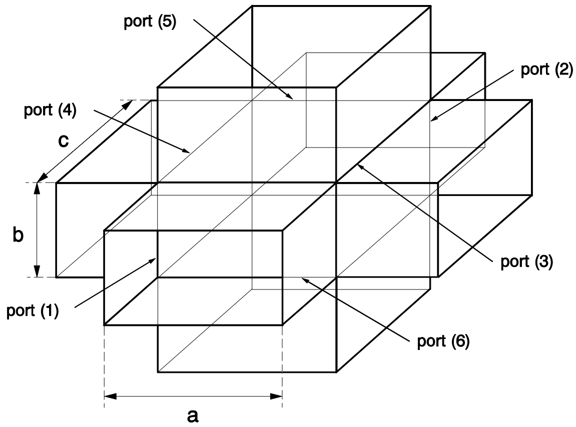


Figure 1: Cubic waveguide junction.

## 2. THE GENERAL C-JUNCTION

The structure under study is the C-junction shown in Fig. 1. The objective is to derive a multimode equivalent network representation, in terms of admittance parameters, with the reference planes for the electrical ports coinciding with the faces of the internal cubic region. According to the theory of cavities [4], such a representation is based on the following generic expression

$$Y_{m,n}^{(\delta,\gamma)} = \sum_{i=1}^{\infty} \frac{j\omega\varepsilon}{\omega^2\mu\varepsilon - K_i^2} \int_{CS(\gamma)} (\mathbf{n} \times \mathbf{e}_n^{(\gamma)}) \cdot \mathbf{h}_i ds$$
$$\int_{CS(\delta)} \mathbf{h}_i \cdot \mathbf{h}_m^{(\delta)} ds + \frac{j}{\omega\mu} \sum_{i=1}^{\infty} \int_{CS(\gamma)} (\mathbf{n} \times \mathbf{e}_n^{(\gamma)}) \cdot \mathbf{g}_i ds$$
$$\int_{CS(\delta)} \mathbf{g}_i \cdot \mathbf{h}_m^{(\delta)} ds \qquad (1)$$

where $\mathbf{h}_i$ and $\mathbf{g}_i$ represent the solenoidal and irrotational modes of the central cubic cavity, respectively, $\omega$ is the angular frequency of the exciting field, $K_i$ the discrete wavenumber of the i-th cavity mode, $\mathbf{n}$ the unitary vector perpendicular to the aperture outward the cavity, and where $\mathbf{h}_m^{(\delta)}$ and $\mathbf{e}_m^{(\delta)}$ represent the m-th vector mode functions of the waveguide in the access port $(\delta)$ (the integration domains CS($\delta$) and CS($\gamma$) indicate the port where the integral is evaluated).

To proceed, we now need to derive the expressions for both the rotational and irrotational vector mode functions of the central cubic cavity, obtained by placing short-circuit terminations in all six ports. This, however, can be very easily accomplished by using as a starting point the standard vector mode functions of a rectangular waveguide (see for instance [5]), and then using the theory described in [6].

With respect to the excitation modes ($\mathbf{e}_n^{(\gamma)}$ and $\mathbf{h}_m^{(\delta)}$) appearing at expression (1), the subindexes $n$ and $m$ stand for $p_2$, $q_2$ and for $p_1$, $q_1$, respectively. The expressions for these vector mode functions must be defined at each access port of the C-junction. Furthermore, for evaluating (1) the term $(\mathbf{n} \times \mathbf{e}_n^{(\gamma)})$ must be computed. However, such term is directly related to the vector mode function $\mathbf{h}_n^{(\gamma)}$, so that it is only necessary to derive expressions for the normalized magnetic field $\mathbf{h}_m^{(\delta)}$. These expressions can be easily obtained for each port from [5].

Then, taking advantage of modal orthogonality, we can easily obtain

$$Y_{m,n}^{(\delta,\delta)} = (-j)\, Y_{0n}^{(\delta)} \cot(\beta_n^{(\delta)} l^{(\delta)})\delta_{m,n} \qquad (2)$$

where

$$\begin{aligned} l^{(1)} &= l^{(2)} = c \\ l^{(3)} &= l^{(4)} = a \\ l^{(5)} &= l^{(6)} = b \end{aligned} \qquad (3)$$

$$Y_{m,n}^{(2,1)} = j\, Y_{0n}^{(1)} \csc(\beta_n^{(1)} c)\delta_{m,n} \qquad (4)$$

$$Y_{m,n}^{(4,3)} = j\, Y_{0n}^{(3)} \csc(\beta_n^{(3)} a)\delta_{m,n} \qquad (5)$$

$$Y_{m,n}^{(6,5)} = j Y_{0n}^{(5)} \csc(\beta_n^{(5)} b)\delta_{m,n} \qquad (6)$$

$$\begin{aligned} Y_{m,n}^{(3,1)} &= \frac{j\omega\varepsilon}{\omega^2\mu\varepsilon - K^2_{p_2,q_2,p_1}} \left(A_x^{(1)} + A_y^{(1)}\right) \\ &\left(B_z^{(3)} + B_y^{(3)}\right) \delta_{q_1,q_2} + \frac{j}{\omega\mu}\left(C_x^{(1)} + C_y^{(1)}\right) \\ &\left(D_z^{(3)} + D_y^{(3)}\right) \delta_{q_1,q_2} \\ A_x^{(1)} &= N_x^{h_i} N_{x_n}^{(1)} Is_{x(p_2,p_2)}^{(1)} Ic_{y(q_2,q_2)}^{(1)} \\ A_y^{(1)} &= N_y^{h_i} N_{y_n}^{(1)} Ic_{x(p_2,p_2)}^{(1)} Is_{y(q_2,q_2)}^{(1)} \\ B_z^{(3)} &= N_z^{h_i} N_{z_m}^{(3)} Is_{z(p_1,p_1)}^{(3)} Ic_{y(q_1,q_1)}^{(3)} \\ B_y^{(3)} &= N_y^{h_i} N_{y_m}^{(3)} Ic_{z(p_1,p_1)}^{(3)} Is_{y(q_1,q_1)}^{(3)} \\ C_x^{(1)} &= N_x^{g_i} N_{x_n}^{(1)} Is_{x(p_2,p_2)}^{(1)} Ic_{y(q_2,q_2)}^{(1)} \\ C_y^{(1)} &= N_y^{g_i} N_{y_n}^{(1)} Ic_{x(p_2,p_2)}^{(1)} Is_{y(q_2,q_2)}^{(1)} \\ D_z^{(3)} &= N_z^{g_i} N_{z_m}^{(3)} Is_{z(p_1,p_1)}^{(3)} Ic_{y(q_1,q_1)}^{(3)} \\ D_y^{(3)} &= N_y^{g_i} N_{y_m}^{(3)} Ic_{z(p_1,p_1)}^{(3)} Is_{y(q_1,q_1)}^{(3)} \end{aligned} \qquad (7)$$

where $Is_{x(i,n)}^{(1)}$, $Ic_{x(i,n)}^{(1)}$, $Is_{y(i,n)}^{(1)}$, $Ic_{y(i,n)}^{(1)}$, $Is_{z(i,m)}^{(3)}$, $Ic_{z(i,m)}^{(3)}$, $Is_{y(i,m)}^{(3)}$ and $Ic_{y(i,m)}^{(3)}$ are simple sine or cosine integrals to be performed on the corresponding variable ($x$, $y$ or $z$ in each case), such as for instance

$$Is_{x(i,n)}^{(1)} = \int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{i\pi x}{a}\right) dx \qquad (8)$$

$$Ic_{z(i,m)}^{(3)} = \int_0^c \cos\left(\frac{m\pi z}{c}\right) \cos\left(\frac{i\pi z}{c}\right) dz \qquad (9)$$

$$Is_{y(i,m)}^{(3)} = \int_0^b \sin\left(\frac{m\pi y}{b}\right) \sin\left(\frac{i\pi y}{b}\right) dy \qquad (10)$$

Note that the coefficients $N_{x_n}^{(1)}$, $N_{y_n}^{(1)}$, $N_{z_m}^{(3)}$ and $N_{y_m}^{(3)}$ present at expression (7) are the normalization factors of the magnetic vector mode functions at the corresponding ports, which can be directly obtained from [5]. On the other hand, the coefficients $N_x^{h_i}$, $N_y^{h_i}$, $N_z^{h_i}$, $N_x^{g_i}$, $N_y^{g_i}$ and $N_z^{g_i}$, also appearing in (7), represent the normalization factors of the solenoidal and irrotational modes of the central cubic cavity which can be simply derived from [6].

With regard to the expressions for the $Y_{m,n}^{(5,1)}$ and $Y_{m,n}^{(5,3)}$ elements of the admittance matrix representation of the cubic junction, they are quite similar to the one outlined in (7) for the $Y_{m,n}^{(3,1)}$ elements, but taking into account the two electrical ports related by the $Y_{m,n}^{(5,1)}$ and $Y_{m,n}^{(5,3)}$ elements.

The remaining elements of the admittance matrix representation of the C-junction can be easily deduced taking into account symmetry properties of such structure, and also from the fact that such junction is reciprocal.

## 3. APPLICATION EXAMPLES

The expressions derived in the previous section for the general C-junction can be easily specialized to particular cases of special interest like, for instance, the H- and the E-plane T-junctions shown in Fig. 2.

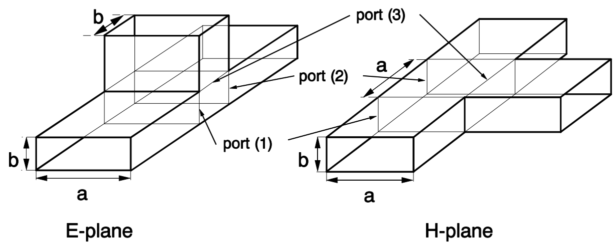


Figure 2: H- and E-plane T-junctions.

The H-plane T-junction can, in fact, be considered as a C-junction (see Fig. 1) with ports $4$, $5$ and $6$ terminated by short-circuits. The same procedure can also be used for the E-plane case. The effect on the actual matrix representation is simply the reduction of the original six by six block structure to the appropriate three by three block structure.

To validate the admittance matrix representations derived, we have computed the scattering parameters of a standard H-plane T-junction implemented with waveguide WR62 ($a = 15.799$ mm, $b = 7.899$ mm). The results obtained are shown in Fig. 3. As we can see, we have an excellent agreement between our results and measurements from [3].

Next we present the scattering matrix results obtained for an E-plane T-junction in waveguides WR62 ($a = 15.799$ mm, $b = 7.899$ mm). The results obtained are shown in Fig. 4 and are again in excellent agreement with the reference measurements [3].

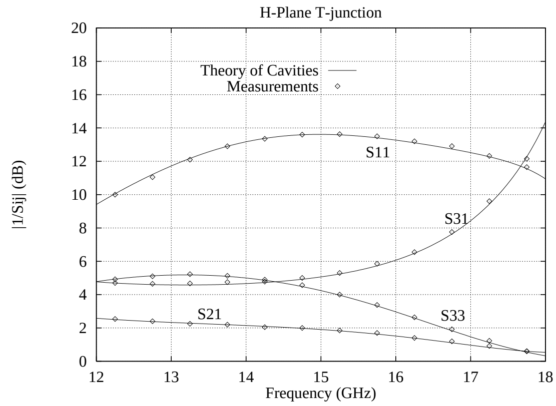


Figure 3: Magnitude of the scattering parameters for a rectangular waveguide (WR62: $a = 15.799$ mm, $b = 7.899$ mm) H-plane T-junction operating in the Ku-band. The solid line indicates our results, the points indicate measurements from [3].

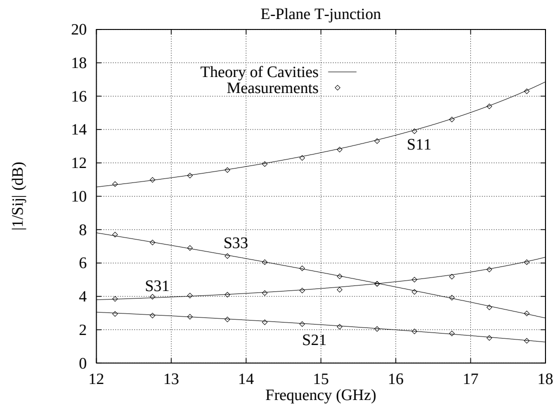


Figure 4: Magnitude of the scattering parameters for a rectangular waveguide (WR62: $a = 15.799$ mm, $b = 7.899$ mm) E-plane T-junction operating in the Ku-band. The solid line represents our results, and the points represent measurements from [3].

In order to obtain the results shown in Fig. 3 and Fig. 4, $20$ modes at each port have been used for both cases. Running the full-wave analysis of each structure has only required 0.04 sec. per frequency point on a CONVEX 1600 machine.

The last application to be considered is the Magic-T junction shown in Fig. 5. In order to characterize this device using the C-junction matrix elements, it is convenient to decompose it in to a four port junction and a step between a square and a standard rectangular waveguide (port number $4$ in Fig. 5). For the central junction, we can again use the C-junction results. The step junction can instead be represented by another admittance matrix representation given in [7].

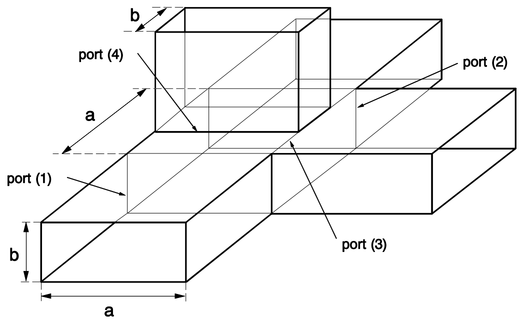


Figure 5: Magic-T junction.

As a validation, we have computed the scattering parameters of a Magic-T in WR62 waveguide ($a = 15.799$ mm, $b = 7.899$ mm). The results obtained can be seen at Fig. 6, and are in very good agreement with the results shown in [3].

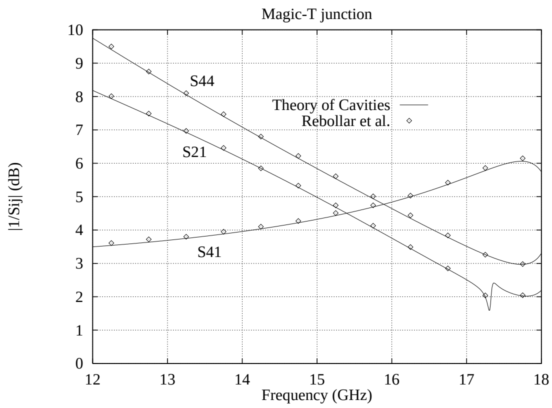


Figure 6: Magnitude of $S_{21}$, $S_{41}$ and $S_{44}$ parameters for a rectangular waveguide (WR62: $a = 15.799$ mm, $b = 7.899$ mm) Magic-T junction operating in the Ku-band. With solid line our results, and with points results shown in [3].

For this case, $20$ modes have been used at each port of the central four-port junction, and $40$ modes have been chosen for the rectangular waveguide. The CPU time for this example, due to its higher complexity, has been of $4.5$ sec. per frequency point on a CONVEX 1600 machine.

## 4. CONCLUSION

In this paper we have developed the multimode admittance matrix representation of a general six-port junction (the C-junction). The value of the representation derived is in that it yelds simple analytical closed-form expressions which are basically independent from frequency. Furthermore, we show how the general expressions can be used to study more common junctions like the E- or H-plane T-junction, and the Magic-T junction. Comparisons with available data are included indicating that the formulation derived is very accurate as well as computationally efficient.

## 5. REFERENCES

[1] J. Esteban and J.M. Rebollar, ”Generalized scattering matrix of generalized two-port discontinuities: application to four-port and nonsymmetric six-port couplers”, IEEE Trans. Microwave Theory Tech.”, vol. MTT-39, no. 10, pp. 1725-1734, Oct. 1991.

[2] F. Alessandri, M. Mongiardo and R. Sorrentino, ”Computer-aided design of beam forming networks for modern satellite antennas”, IEEE Trans. Microwave Theory Tech.”, vol. MTT-40, no. 6, pp. 1117-1127, June 1992.

[3] J.M. Rebollar, J. Esteban and J.E. Page, ”Full-wave analysis of three and four-port rectangular waveguide junctions”, IEEE Trans. Microwave Theory Tech.”, vol. MTT-42, no. 2, pp. 256-263, Feb. 1994.

[4] K. Kurokawa, *An introduction to the theory of microwave circuits*, Academic Press, New York and London, 1969.

[5] N. Marcuvitz, *Waveguide Handbook*, Peter Peregrinus Ltd., IEE Electromagnetic Waves Series 21, London, UK, 1986.

[6] R.E. Collin, *Field theory of guided waves*, second edition, IEEE Press, USA, 1991.

[7] A. Alvarez, G. Connor and M. Guglielmi, ”New simple procedure for the computation of the multimode admittance or impedance matrix of planar waveguide junctions”, IEEE Trans. Microwave Theory Tech., vol. MTT-44, no. 3, pp. 413-418, March 1996.